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OBSERVATIONS ON USE OF THE J INTEGRAL TO DETERMINE PLANE-STRAIN FRACTURE TOUGHNESS FROM SUBSIZED SPECIMENS OF A TITANIUM 6Al-4V ALLOY

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J-integral procedures were used to obtain resistance curves of J versus crack extension from fatigue-precracked specimens of a titanium 6Al-4V alloy. Specimens of various geometries were employed, all in three-point bending. Crack extensions were delineated by heat tinting. The value of the J integral at the initiation of crack extension, J_{Ic}, was determined from the resistance curves and compared with a valid K_{Ic} value for this material. Comparison was made of the results obtained with the approximation equation $J = 2A/Bb$ to those obtained with the more cumbersome compliance technique in which the analytical plane-stress solutions of

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Bucci et al. were used. The range of ratios of crack length to specimen width was examined over which $J = 2A/Bb$ holds, as was the decrease in crack extension at the experimental limit load with increase in ratio of specimen thickness to uncracked ligament. Variation in J_{Ic} from specimens cut from different positions through the plate thickness was also explored.

CONTENTS

OBSERVATIONS ON USE OF THE J INTEGRAL TO DETERMINE PLANE-STRAIN FRACTURE TOUGHNESS FROM SUBSIZED SPECIMENS OF A TITANIUM 6Al-4V ALLOY

INTRODUCTION

A growing body of experimental evidence [1 − 5] supports the critical value of Rice's J integral [6, 7] J_{Ic} as a criterion for the initiation of crack extension in elastic-plastic fracture. Recent J-integral studies demonstrate success with an unload/heat-tint, resistance-curve technique to determine J_{Ic} in steel [3], titanium [4], and aluminum [5] alloys. The purpose of this report is to provide a comprehensive summary of results to date from a study of J_{Ic} determination with a titanium 6Al-4V alloy.

In this work eight different types of fatigue-precracked, three-point-bend specimens have been tested. For several of these, resistance curves of J vs crack extension (Δa) have been obtained by heat tinting multiple specimens of a given type which have been unloaded from different points on the respective diagram of load (P) vs load point displacement (δ). For purposes of comparison, evaluation of the J integral has been made by two methods: (a) the Begley and Landes compliance calibration technique [1, 2], using the plane-stress plastic-zone-size corrected solutions of Bucci et al. [8] which simulate well the experimental P-vs-δ traces obtained from the precracked specimens; and (b) the approximation equation proposed independently by Rice et al. [9] and Srawley [10]:

$$J = \frac{2A}{Bb}, \tag{1}$$

where A is the area under the P-vs-δ curve at the point of interest, B is specimen thickness, and b is the uncracked ligament. To define the point of initiation of crack extension Δa_c and hence J_{Ic}, it is necessary to select a criterion, perhaps somewhat arbitrarily at this point. The one used in this work was proposed recently by Paris [11], namely that Δa_c be defined by the largest amount of actual crack extension (1%) permitted in the smallest allowable K_{Ic} specimen:

$$\Delta a_c = 0.025 \frac{EJ_{Ic}}{\sigma_{ys}^2 (1 - \nu^2)}, \tag{2}$$

where E is Young's modulus, σ_{ys} is the uniaxial yield strength, and ν is Poisson's ratio. Values of J_{Ic} obtained in this study are compared to a valid K_{Ic} value for this alloy via

$$J_{Ic} = G_{Ic} = \frac{K_{Ic}^2 (1 - \nu^2)}{E}, \tag{3}$$

Note: Manuscript submitted June 28, 1974.

where material constants G_{Ic} and K_{Ic} are critical values of the crack extension force and stress-intensity factor respectively.

The specimen type of least thickness is used to examine for possible variation in J_{Ic} as a function of position through the plate thickness. In another series of tests, the range of ratios of crack length to specimen width (a/W) is explored over which Eq. (1) holds.

EXPERIMENTAL PROCEDURE

Material and Specimens

All specimens were cut from a 1-in.-thick plate of mill-annealed Ti-6Al-4V alloy with the chemical composition and mechanical properties given in Table 1. Light photomicrographs in Fig. 1a reveal a microstructure consisting of elongated primary α grains dispersed in an α-β Widmanstatten (basketweave) matrix. Extensive crossrolling is evident from these micrographs and seems to be reflected in the tensile properties, determined with standard 0.505-in.-diameter specimens. The yield strength in both the longitudinal (L) and transverse (T) directions is 124 ksi; Young's modulus is 18.55×10^3 ksi in the T direction. The mode of crack extension in this alloy is microvoid coalescence, as illustrated by the replica electron micrograph of Fig. 1b.

Cross-sectional geometries of the eight types of three-point-bend specimens used, A through H, are presented in Fig. 2 with a list of dimensions. Thicknesses (B) range from 0.250 to 1.000 in., widths from 0.658 to 1.500 in., a_0/W ratios from 0.313 to 0.745, and B/b ratios from 0.61 to 2.99. All specimen types, with one exception, were machined from the plate midthickness, that is, with equal amounts of metal removed from the plate surfaces relative to the B dimension. In the case of type E, subscripts are used to designate position of the specimen relative to plate thickness, that is, type E_c from the center or plate midthickness vs type E_s cut from as near the plate surface as possible. All specimens were fatigue precracked at levels of stress-intensity factor permitted by ASTM E399-72 [12], with crack orientations all in the TL direction. These specimen types exhibited, to varying degrees, loading behavior characteristic of the lower end of the elastic-plastic regime. That is, all exhibited limit loads which were substantially less than those expected for the fully plastic state, which may be approximated for pure bending by [8, 13]

$$P_L = 1.456\ \sigma_{ts} \frac{B}{S} (W - a)^2, \tag{4}$$

where σ_{ts} is the uniaxial tensile strength and S is the span length. Moreover, behavior of the specimen type with the most highly constrained crack (type A) appeared to be only marginally "invalid" with respect to ASTM E399-72, as only one specimen of three of this type examined for K_Q provided a valid K_{Ic} value of 59.7 ksi$\sqrt{\text{in.}}$, as noted in the stress-intensity-factor data of Table 1.

Table 1
Alloy Composition and Mechanical Properties

CHEMICAL COMPOSITION
(WEIGHT PERCENT)

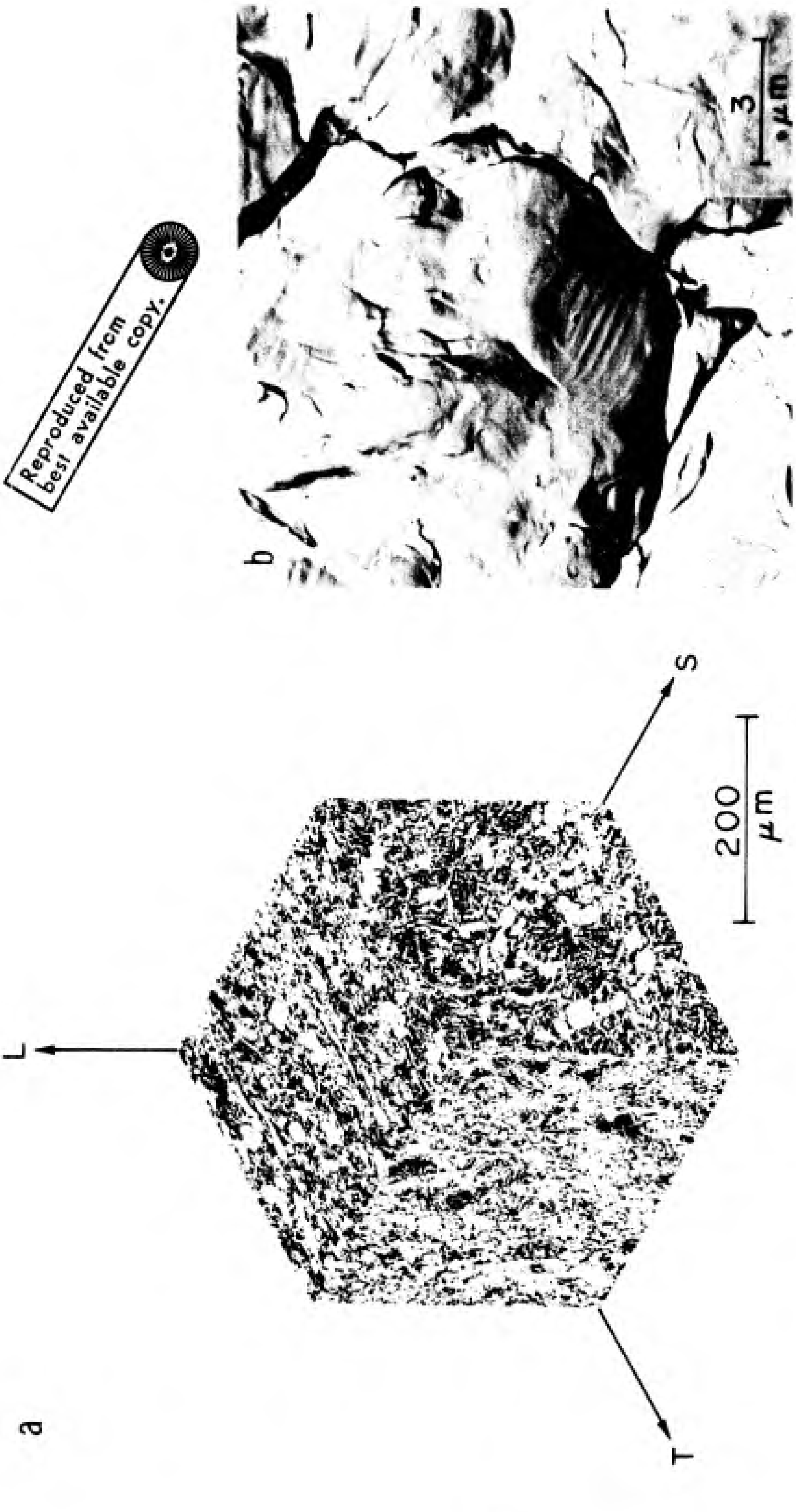
Al	V	Fe	C	N	H	O	Ti
6.0	4.1	0.05	0.023	0.008	0.005	0.06	Bal.

TENSILE PROPERTIES

0.2% Offset Yield Strength		Tensile Strength		% Reduction Area	% Elongation	Young's Modulus	
ksi	MPa	ksi	MPa			$\times 10^3$ ksi	GPa
Transverse direction (T)							
124.4	868.3	133.8	933.9	39.5	13.5 (in 2 in.)	18.55	129.5
Longitudinal direction (L)							
124.1	866.2	130.5	910.9	39.5	16.5 (in 1.4 in.)	18.56	129.5

STRESS INTENSITY FACTOR DATA

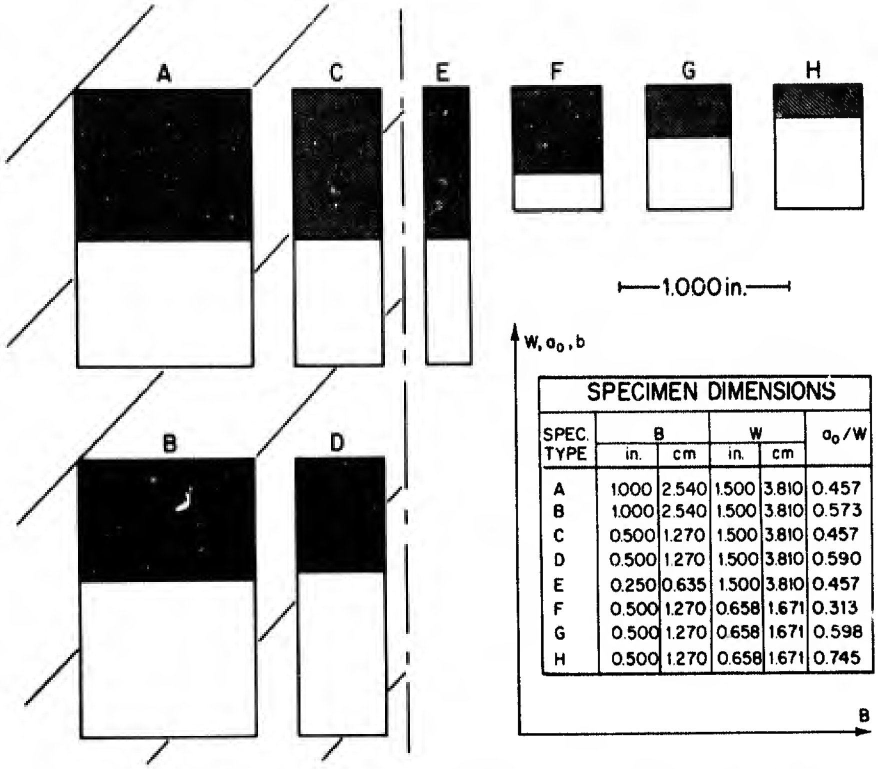
Specimen Type		$\frac{P_{max}}{P_Q}$	K_Q ksi-$\sqrt{in.}$	2.5 $(K_Q/\sigma_{ys})^2$ <B, a, W − a	$K_Q = K_{Ic}$
A	(a)	1.12	59.7	Yes	No
	(b)	1.09			Yes
	(c)	1.11			No
C	(a)	1.23			No



(a) Light optical photomicrograph of microstructure;
etched with Kroll's reagent

(b) Replica electron fractograph
of dimpled rupture

Fig. 1 — Definition of alloy microstructure and mode of crack extension



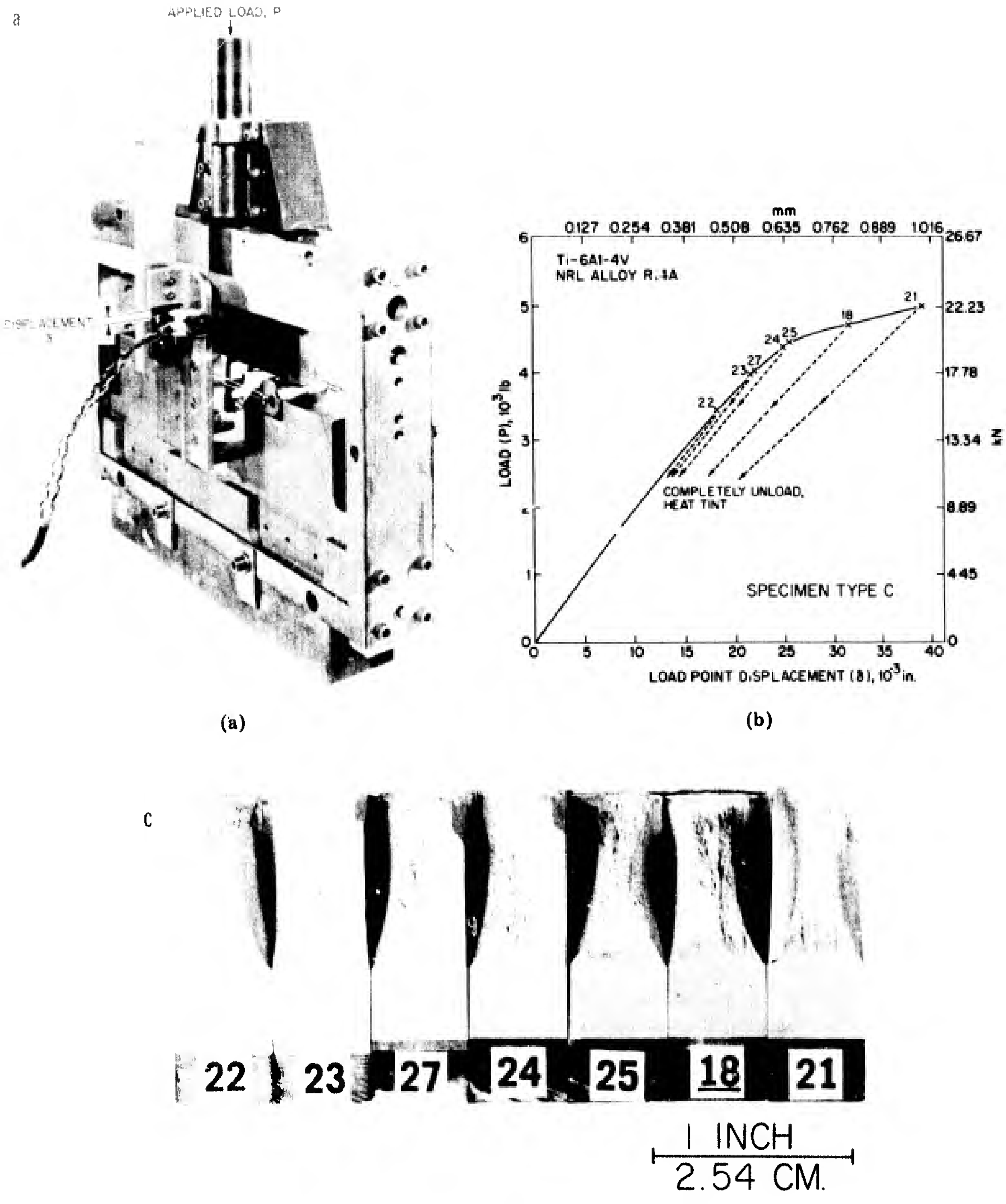
SPECIMEN DIMENSIONS					
SPEC. TYPE	B		W		a_0/W
	in.	cm	in.	cm	
A	1.000	2.540	1.500	3.810	0.457
B	1.000	2.540	1.500	3.810	0.573
C	0.500	1.270	1.500	3.810	0.457
D	0.500	1.270	1.500	3.810	0.590
E	0.250	0.635	1.500	3.810	0.457
F	0.500	1.270	0.658	1.671	0.313
G	0.500	1.270	0.658	1.671	0.598
H	0.500	1.270	0.658	1.671	0.745

Fig. 2 — Cross-sectional views of specimen types and list of the dimensions. Shaded portion of the cross section indicates ligament b × B.

Test Procedure

All specimen types were tested in the fixture shown in Fig. 3a. A clip gage was used to measure δ as illustrated; a supplementary gage can be seen to span the crack mouth opening in accord with ASTM E399-72. The span between rollers was S = 5.975 in. All tests were conducted in room-temperature air.

To determine the point on a P-vs-δ diagram at which crack extension initiated, multiple specimens of a given type were loaded to various points on the respective P-vs-δ diagram (anywhere from the incidence of nonlinearity to maximum load), followed by complete unloading, as illustrated in Fig. 3b, for specimens of type C. Specimens were then heat tinted in a circulating-air furnace at 600°F (589 K) for 2 hrs and broken open for examination. Crack extensions corresponding to the points of unloading in Fig. 3b are shown in the photograph of Fig. 3c. Initial fatigue-precrack length was measured according to ASTM E399-72, as was the crack extension delineated by heat tinting; that is, the amount of crack extension was taken as an average of that measured at the quarterpoints of specimen thickness. Inasmuch as the initiation of crack extension is a heterogeneous nucleation process, this is admittedly an arbitrary measure of crack extension and therefore should be kept in mind as a potential source of scatter in results. Uncertainty in individual measurements is $\approx \pm 0.001$ in.



(a) (b)

(c)

Fig. 3 — Test procedure: multiple specimens of each type were loaded in the three-point-bend test fixture shown in (a) to various points on the respective P-vs-δ diagram as shown in (b); then they were unloaded and heat tinted to reveal crack extensions as shown in (c). The case illustrated is for specimen type C, with individual specimen numbers identified in (b) and (c).

In one of the ways used to evaluate the J integral, the Begley and Landes technique was used to obtain a compliance calibration from P-vs-δ diagrams generated for several a/W ratios from plane-stress plastic-zone-size corrected solutions of Bucci et al. These P-vs-δ traces were integrated graphically using the trapezoidal rule, with increments of 0.005-in. displacement. These solutions simulate well the experimental P-vs-δ traces obtained from the precracked specimens, as illustrated in Fig. 4 for specimen types A through D; notable deviations at some of the greatest displacements are attributable to crack extension in the precracked specimens. In using Eq. (1) to evaluate the J integral, it is appropriate to note that A has been taken to be area under the actual experimental P-vs-δ trace, minus the component owing to the test fixture, in accord with the Srawley formulation.

RESULTS AND DISCUSSION

Comparison of Resistance-Curve Data

Resistance-curve data of J vs Δa for specimen types A through D are presented in Figs. 5a and 5b, for which J was evaluated from the compliance calibration technique and Eq. (1) respectively. Curves sketched for each specimen type in Fig. 5a are shown as solid lines in Fig. 5c for comparison with the respective dashed curves from Fig. 5b. At lower values of Δa, the scatter in data defined by the solid lines overlaps quite well that from the dashed lines, although near $\Delta a = 0$ there is a tendency toward slightly lower J values as calculated from Eq. (1). On the other hand, at the higher values of Δa, the dashed curves indicate notably greater J values, as maximum load (P_{max}) is approached, than do the respective solid lines, particularly for specimen types C and D. This may be a reflection of Landes and Begley's prediction that J values computed by Eq. (1) would be overestimated, particularly at greater Δa, owing to the influence of crack growth effects on A [3]. By comparison, the compliance technique would not be expected to so overestimate J.

To infer levels of J_{Ic} from Figs. 5a and 5b, Δa_c can be estimated from Eq. (2) to be approximately 6×10^{-3} in., by using the mean extrapolated value of J for $\Delta a = 0$ in Fig. 5, or from the value of K_{Ic} noted earlier. From Fig. 5a, it follows that $J_{Ic} = 188 - 235$ in. lb/in.^{2} or 211 ± 24 in. lb/in.^{2}; this compares with $J_{Ic} = 158 - 209$ in. lb/in.^{2} or 184 ± 26 in. lb/in.^{2} inferred from Fig. 5b. This amounts to a variation in J_{Ic} of $<\pm 12\%$ from Fig. 5a and $<\pm 15\%$ from Fig. 5b. These correspond to variations in K_{Ic} of $<\pm 6\%$ and $<\pm 7\%$ respectively. This is substantially less variation than might well be found in K_{Ic} testing of mill-annealed titanium alloy plates [14]. The question as to which of the two mean values, $J_{Ic} = 184$ or 211 in. lb/in.^{2}, is the more accurate is a moot point. The value of K_{Ic} cited in Table 1 translates to $J_{Ic} = 170$ or 192 in. lb/in.^{2}, depending on whether the factor $(1 - \nu^2)$ should be included or not in Eq. (3) [15, 16]. Moreover, it can be argued that the quoted K_{Ic} value in fact may be lower than the average value for the plate material, inasmuch as two out of three K_Q tests failed to provide a valid K_{Ic} determination.

Specimen-Size Analysis

It has been estimated that "legal" J_{Ic} determinations can be obtained with specimens of limiting dimensions [3, 17]

$$a, B, b > \alpha \frac{J_{Ic}}{\sigma_{flow}}, \tag{5}$$

with $\alpha = 50$ and σ_{flow} taken as the mean of σ_{ys} and σ_{ts}. For a level of $J_{Ic} = 195$ in. lb/in.^{2}, this means for the present alloy that a, B, and b must exceed about 0.08 in. This value is exceeded by the respective dimensions of all specimen types A through H.

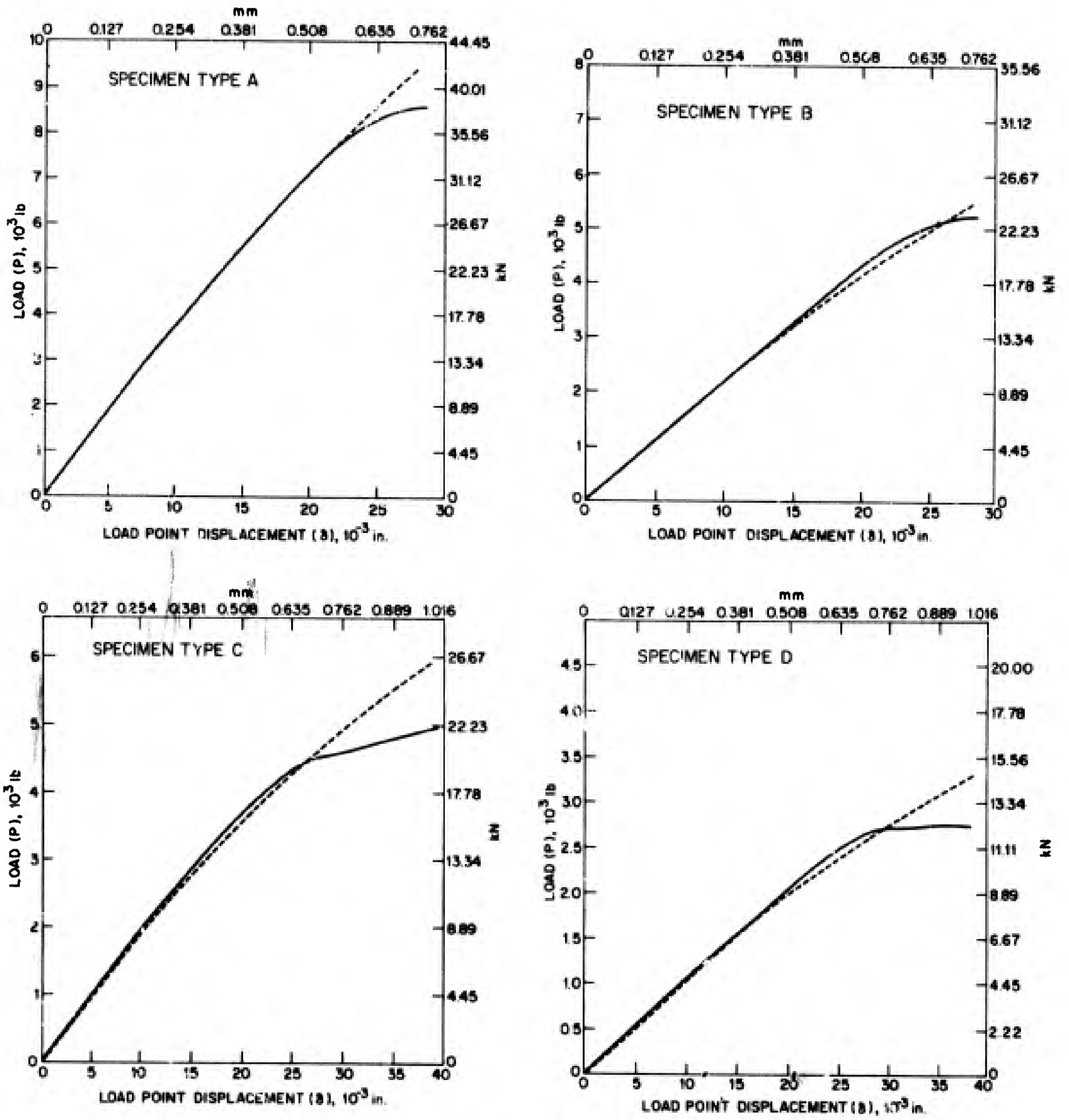
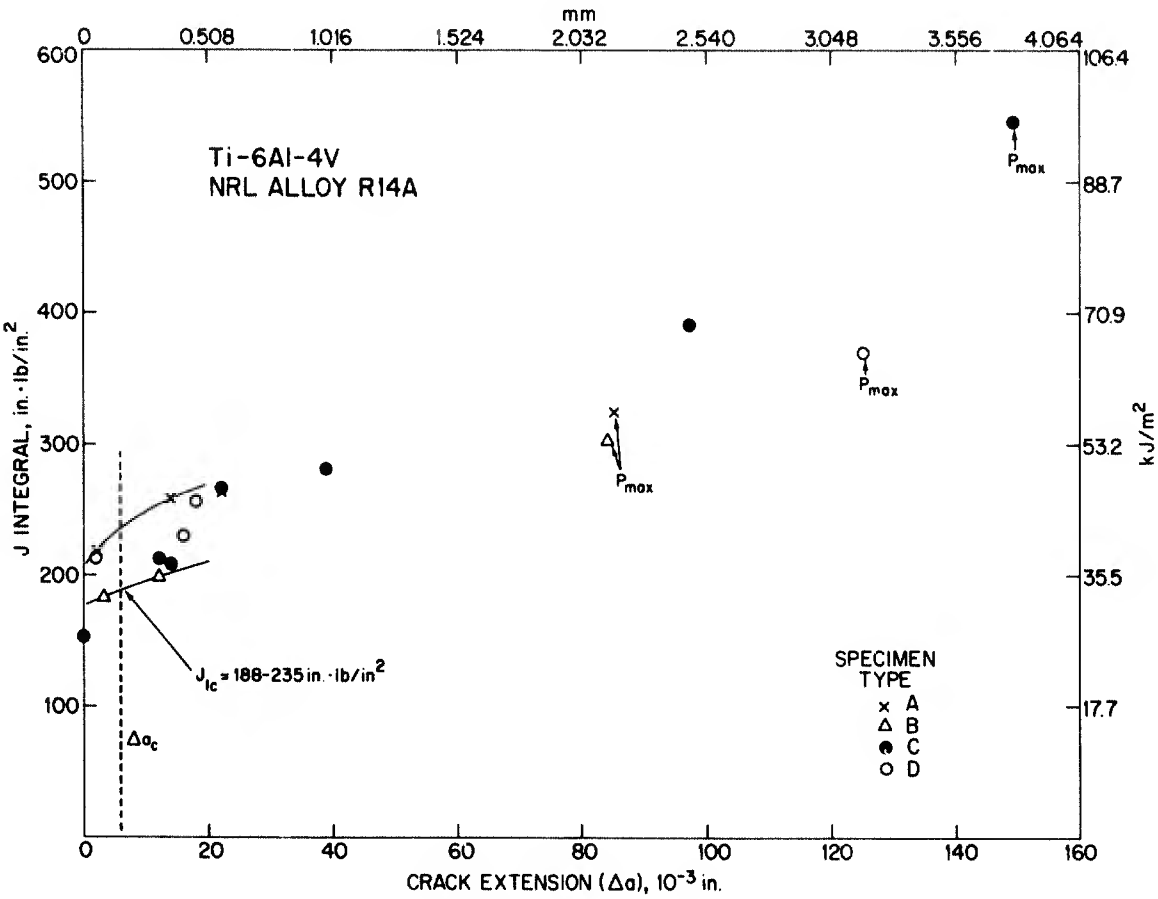
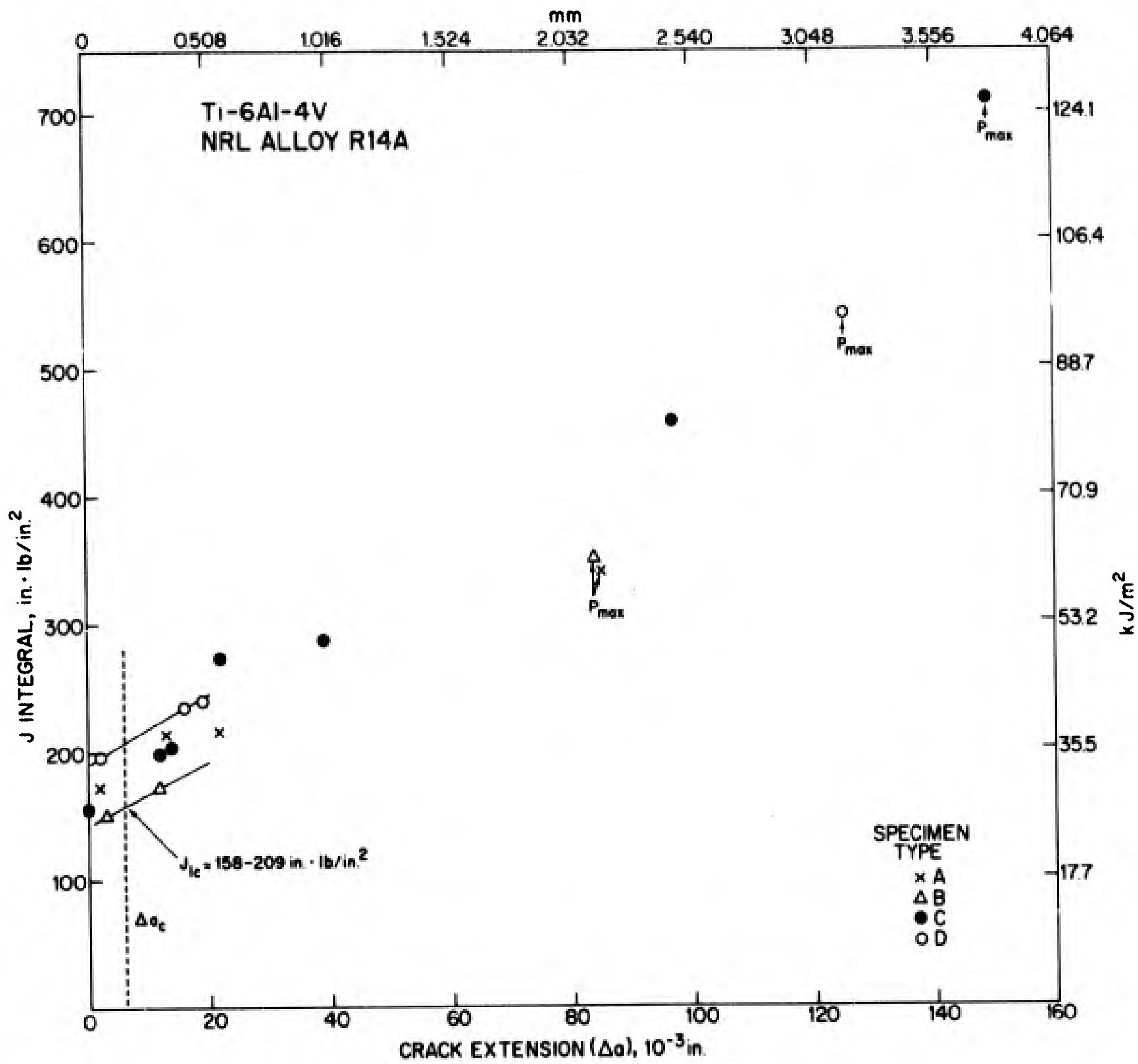


Fig. 4 — Comparison of P-vs-δ diagrams obtained from precracked specimens to those generated from the plane-stress plastic-zone-size corrected solutions of Bucci et al. [8]



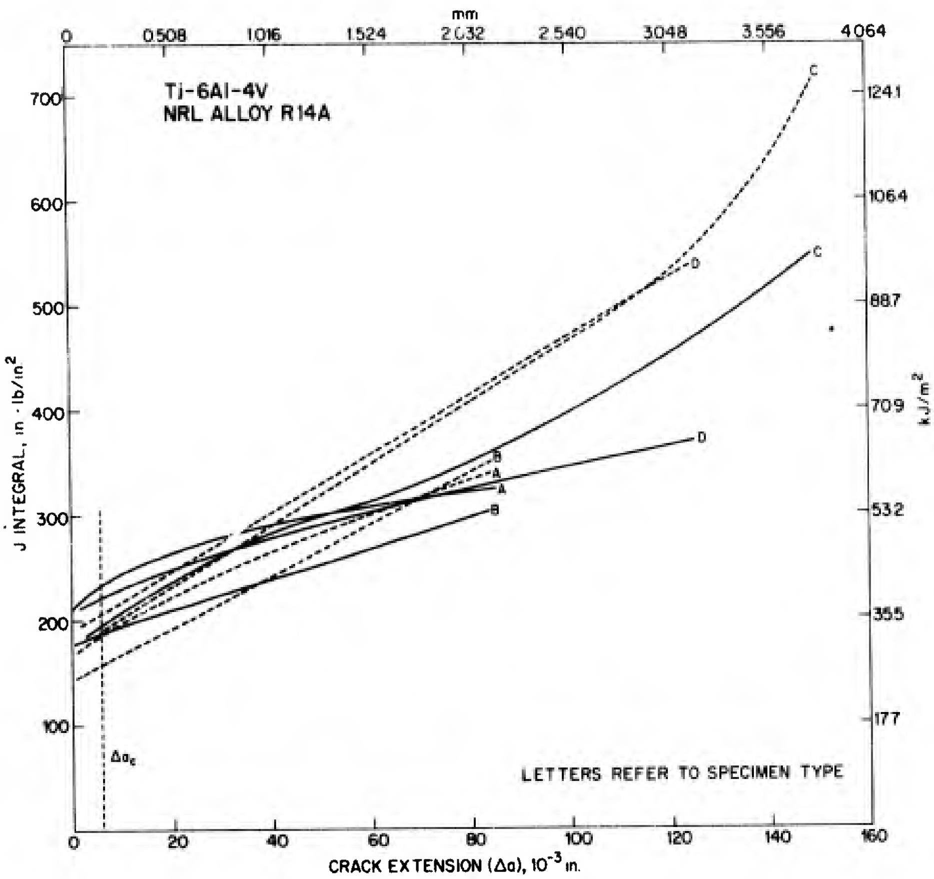
(a)

Fig. 5 — Resistance curves of J vs Δa for specimen types A through D, from which J_{Ic} is defined at Δa_c; (a) Data for J evaluated by the compliance calibration technique, (b) Data for J evaluated from $J = 2A/Bb$, (c) Comparison of the resistance curves from data in Figs. 5a and 5b.



(b)

Fig. 5 (Continued) — Resistance curves of J vs Δa for specimen types A through D, from which J_{Ic} is defined at Δa_c; (a) Data for J evaluated by the compliance calibration technique, (b) Data for J evaluated from J = 2A/Bb, (c) Comparison of the resistance curves from data in Figs. 5a and 5b.



(c)

Fig. 5 (Continued) — Resistance curves of J vs Δa for specimen types A through D, from which J_{Ic} is defined at Δa_c; (a) Data for J evaluated by the compliance calibration technique, (b) Data for J evaluated from J = 2A/Bb, (c) Comparison of the resistance curves from data in Figs. 5a and 5b.

Variation of J_{Ic} With Position Through the Plate Thickness

Specimens E_c and E_s were unloaded at displacements near the initiation point and heat tinted. Results are plotted in Fig. 6a relative to the data scatter band obtained near initiation in Fig. 5a, for specimen types A through D with J evaluated by the compliance method. A substantial difference in J values is evident between the two specimens: for Δa = 12 and 13 mils respectively, specimen E_c exhibits a level of J = 256 in. lb/in.^{2}, whereas J = 159 in. lb/in.^{2} for specimen E_s.

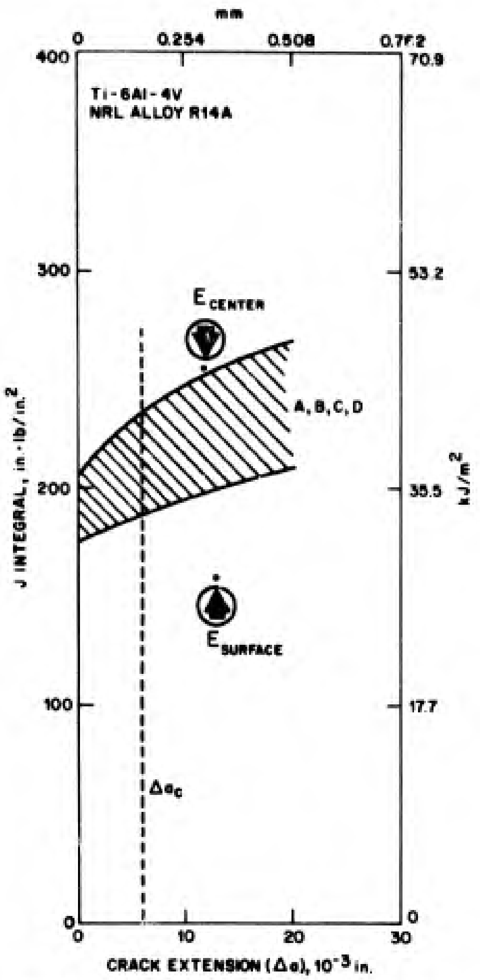
As might be expected, replica electron fractographs of actual heat-tinted crack extension reveal significantly larger dimples in the region of greater toughness, namely the plate midthickness (Fig. 6b), than appear in the region near the plate surface (Fig. 6c). (Note that heat tinting has done little to obscure fractographic features in this alloy.) Though the microstructure of this alloy is relatively uniform for a mill-annealed plate of Ti-6Al-4V, the primary α phase was somewhat greater in percentage and more elongated in shape at the plate midthickness; the grain size of the α-β Widmanstatten matrix was also somewhat finer there than near the surface. On the other hand, Rockwell C hardness measurements (HRC) made through the plate thickness showed little variation. On the face normal to the L direction, HRC varied from ~31 at the center to ~32 at the surface; however, on the face normal to the T direction, HRC was higher in the center (~33.5) than near the surface (~32.0).

Effect of a/W and B/b Ratios on J_{Ic} Determination

Specimen types F, G, and H were tested to explore the range of a/W ratios for which J may be calculated via Eq. (1); Srawley [10] has suggested that the range should extend from 0.2 to 0.95. Specimens of these three types were unloaded near the initiation point and heat tinted. Results are plotted in Fig. 7 relative to the data scatter band obtained near initiation in Fig. 5b for specimen types A through D. For specimen type F (a/W = 0.313), the J value calculated from Eq. (1) is obviously too high, namely 306 in. lb/in.^{2} at Δa = 3 mils. Though for specimen type G (a/W = 0.598) the J value appears reasonable, that for specimen H is obviously too low with J = 148 in. lb/in.^{2} at Δa = 13 mils.

In view of Rice's derivation of Eq. (1), the result for specimen type F might not be unexpected; however, for a deeply cracked specimen such as type H, it is. From Table 2, which summarizes J_{Ic} data obtained via Eq. (1) for the different specimen types, it is evident that specimen type H has the smallest dimension b (0.167 in.). Though this dimension is about twice the limiting value of 0.08 in. suggested by $\alpha = 50$ in Eq. (5), perhaps this limiting value is too low for the present case; that is, maybe $\alpha \approx 100$ or greater. It is pertinent to note that for the case of bend bars of a rotor steel, Landes and Begley [2] noted a decrease in apparent J_{Ic} for a specimen with $b < 50\ J_{Ic}/\sigma_{flow}$. Similarly for the case of specimen type F, it is possible that specimen size limitation could be a factor in the result, since type F has the smallest dimension a of all types examined, namely 0.206 in.

In the interest of developing a single specimen test for J_{Ic}, as opposed to the multiple specimens required by the unload/heat-tint, resistance-curve technique, Corten has proposed [18] that J_{Ic} can be obtained from a single specimen if $B/b \geqslant 2$ and if $P_{max}/P_L \geqslant 0.85$; that is, under these conditions, it is proposed that $\Delta a_{P_{max}} \to \Delta a_c$. For each specimen type listed in Table 2, the B/b ratio is given as well as the experimentally observed P_{max}, the ratio P_{max}/P_L, and the value of J corresponding to P_{max}, namely $J_{P_{max}}$. Also included are values of crack extension found at P_{max}, $\Delta a_{P_{max}}$, for comparison with Δa near the initiation point, Δa_c. None of the specimen types exhibit $P_{max}/P_L \geqslant 0.85$, as they range from a low of 0.40 for type A to a high of 0.56 for type H. However, it is apparent that $\Delta a_{P_{max}}$ is an inverse function of B/b, with $\Delta a_{P_{max}}$ ranging from a high of 0.149 in. at the lowest B/b = 0.61 to a low of Δa = 0.013 in. at the highest B/b = 2.99; this is readily seen from the



(a) J-integral data evaluated by the compliance calibration technique and plotted relative to data for specimen types A through D near initiation

(b) Replica electron fractograph of crack extension at the plate midthickness after heat-tinting

(c) Replica electron fractograph of crack extension near the plate surface after heat-tinting

Fig. 6 — Differential in toughness between the plate midthickness region and region near the plate surface, as revealed from specimen types E_c and E_s respectively. Dimple size is notably larger in the region corresponding to the position of type E_c than that of E_s, shown in (b) and (c) respectively.

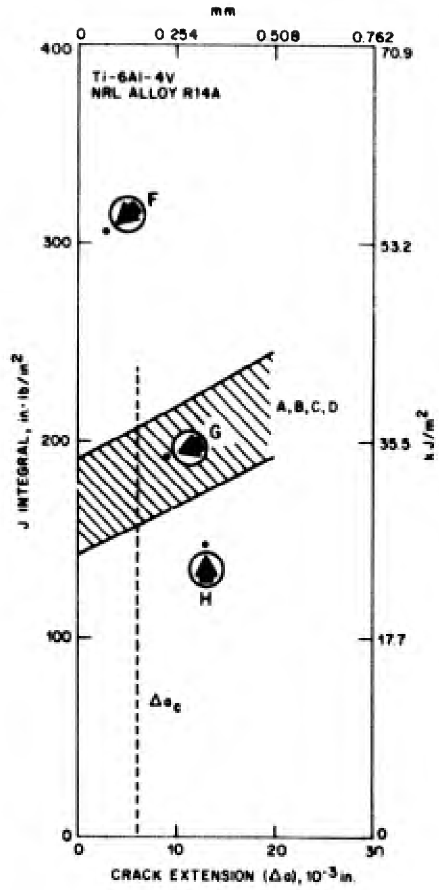


Fig. 7 — Data of J vs Δa for specimen types F through H, with J computed from $J = 2A/Bb$ and plotted relative to data for specimen types A through D near initiation

Table 2
Summary of J-Integral Data for the Various Specimen Types, with J Evaluated from $J = 2A/Bb$

Specimen Type	a_0/W (nom.)	J_{Ic} (approx.) (in. lb/in.^{2})	Δa_c (approx.) (10^{-3} in.)	$\Delta a_{P_{max}}$ (10^{-3} in.)	P_{max} (lb)	$\frac{P_{max}}{P_L}$	b (in.)	B/b	$J_{P_{max}}$ (in. lb/in.^{2})
A	0.457	186	6	85	8600	0.40	0.815	1.23	341
B	0.573	158	6	84	5585	0.44	0.640	1.56	351
C	0.457	190	6	149	4975	0.46	0.815	0.61	711
D	0.590	206	6	125	2790	0.45	0.615	0.81	542
F	0.313	306	3	81	1548	0.47	0.452	1.11	472
G	0.598	192	9	<45	623	0.55	0.265	1.89	<274
H	0.745	148	13	13	257	0.56	0.167	2.99	148

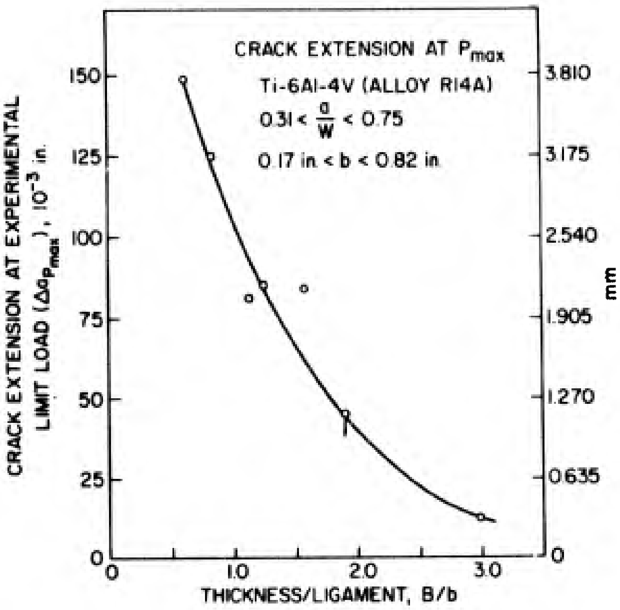


Fig. 8 — Decrease in crack extension at the experimental limit load ($\Delta a_{P_{max}}$) with increase in ratio of specimen thickness to uncracked ligament (B/b)

graphical presentation of Fig. 8. Values of Jp_{max} vary similarly with B/b. Though these trends support Corten's basic idea, unfortunately it may be difficult to design "legal" J_{Ic} specimens of this alloy to attain $P_{max}/P_L > 0.85$. Comparison of types D and G, specimens of equal B and $a/W \approx 0.6$, shows that reduction of W from 1.5 to 0.66 in. led to an increase of P_{max}/P_L from 0.45 to only 0.55; however, W could not be so reduced much further if the size limitation on a and b is indeed ≈ 0.2 in. for this alloy (if $\alpha = 100$ in Eq. (5)).

SUMMARY

Tests were conducted to determine J_{Ic} from resistance curves of J vs crack extension obtained from fatigue-precracked specimens of a titanium 6Al-4V alloy. Three-point-bend specimens of eight geometries were employed, with crack extensions delineated by heat tinting. Findings from this work include:

- Over the range a/W = 0.45 to 0.60, the determination of J_{Ic} obtained with J computed from the equation J = 2A/Bb agrees quite well with that obtained by evaluating J via the compliance calibration technique; moreover, these determinations of J_{Ic} are in good agreement with a valid K_{Ic} value for this alloy.

- At the greater crack extensions, as experimental limit load is approached, resistance curves obtained with J computed as J = 2A/Bb are notably higher than those obtained with J evaluated by the compliance method.

- In exploring the range of a/W over which $J = 2A/Bb$ is applicable, erratic results were obtained at ratios of 0.31 and 0.75. Though the former might be interpreted to be outside the range of applicability, the latter may imply that the specimen-size limitation defined by a, B, $b > \alpha\, J_{Ic}/\sigma_{flow}$ was violated at a surprisingly high level of $\alpha \approx 100$. Further work is suggested to determine whether α might indeed be this high for titanium alloys in general.

- Significant variation in J_{Ic} was found from specimens machined from different positions through the plate thickness. This finding has serious implications regarding any future standard method of J_{Ic} testing: If J_{Ic} results obtained from relatively small, thin specimens are to be used to estimate toughness of a thick plate from which they are cut, such specimens should be made at multiple positions through the plate thickness. If the variance of J_{Ic} with position exceeds some specified percentage, the estimate for the thick plate should not be "legal," at least until some rational averaging procedure is adopted.

- Crack extension at maximum load was found to vary inversely with B/b. For B/b ratios ranging from 0.61 to 2.99, $\Delta a_{P_{max}}$ decreased from 0.149 to 0.013 in. Values of $J_{P_{max}}$ similarly decreased with increasing B/b ratio.

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